

OPTIMIZED EFFICIENCY MAPS AND NEW CORRELATION FOR PERFORMANCE PREDICTION OF ORC BASED ON RADIAL TURBINE FOR SMALL-SCALE APPLICATIONS

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ABSTRACT

The expander is considered as the most critical component of the ORC. Radial inflow turbine exhibits unique advantages of high efficiency, compact structure and light weight compared to the axial turbine when employed in the small-scale applications such as distributed CHP systems. In most of the ORC studies the turbine efficiency is assumed as a constant input for the optimization of cycle without assuring that the specified turbine efficiency can be achieved by the imposed thermodynamic conditions. In addition, atypical properties of the high-density working fluid and the near-critical operating condition of the ORC requires the turbine design procedure and parameters to be customized for the ORC. This study presents the optimization of a radial ORC turbine for maximum efficiency using mean-line modelling and genetic algorithm (GA). In contrast to the previous studies, real gas equation of state and the most advanced and recent loss models are employed in the code to capture the non-ideal behaviour of the working fluid. The optimized turbine efficiency is achieved by the GA for a wide range of operating conditions and for four organic fluids (R123, R245fa, R1233zd and isobutane). Such results are presented through new generalized maps based on the non-dimensional parameters as the flow and loading coefficients, specific speed and specific diameter. Using regression analysis a new correlation for the turbine efficiency is also presented. These new maps and the correlation are preliminary steps toward improving the previous constant turbine efficacy assumption and have great potential to be integrated with the general optimization methods of the ORC.

1. INTRODUCTION

Recently, ORC technology has received a growing attention due to its competitive performance and flexibility for small to medium scale power generation applications. Majority of studies about the organic Rankine cycle (ORC) are mainly devoted to the methods of selecting appropriate fluid for different applications (Aleksandra and Władysław, 2007, Al-Sulaiman *et al.*, 2011, Dolz *et al.* 2012, Rayegan and Tao, 2011) or to the thermodynamic optimization of the cycle based on a range of performance indexes such as the thermal and exergy efficiencies, net power output or heat exchangers area (Rashidi *et al.* 2011, Hettiarachchi, 2007, Wang *et al.*, 2013). On the other hand, little attention has been paid to the design and performance characteristics of the expander. In all above studies that seek the optimum cycle parameters, fixed values are assigned to the turbine efficiency for a wide range of operating conditions and for various working fluids while ignoring the feasibility that the turbine is capable of achieving these efficiencies in practice. In addition, due to the specific thermodynamic properties of the organic fluids (i.e. large molecular weight and low speed of sound), the turbine can behave differently from those that operate with steam or other gases and needs a special design. Moreover, operating near critical point of organic fluids makes the ideal gas models unsatisfactory and requires the real gas formulation of the turbine expansion. Considering these facets, the use of conventional generalized performance correlation charts such as (Rodgers and Geiser 1987, Balje 1981, Chen and Baines 1994) can become inaccurate when used for performance prediction of turbines operating with organic fluids. In this study a new approach for performance prediction of ORC based

on radial turbine is presented that is suitable for small scale applications such as distributed combined heat and power (CHP) systems with power capacity of up to 50kW. In this novel approach the conventional radial turbine performance maps are updated and optimized based on the real gas properties of organic fluids, genetic algorithm (GA) optimization technique and the most advanced and recent loss models by Aungier (2006). A mean-line model for design, analysis and performance prediction of radial turbines is developed in the engineering equation solver (EES) platform and directly embedded with the ORC system code. Such model allows for replacement of the fixed turbine efficiency by an interactive value that is calculated based on the ORC thermodynamic conditions and working fluids properties and assures that the optimized turbine efficiencies are achievable in practice. Such turbine efficiencies are correlated through new maps against loading and flow coefficients, specific speed and specific diameter for four different organic fluids as R123, R245fa, R1233zd and isobutane. Furthermore, a new correlation for turbine efficiency is presented by employing the regression analysis, though, it is considered to be a preliminary one as it requires to be validated against ORC experimental data which are very limited in the literature and also be extended to other working fluid families such as ethers and siloxanes.

2. NON-DIMENSIONAL PARAMETERS

Turbomachines require a set of non-dimensional parameters that can readily describe the machine's overall performance for assessment, selection and comparison. Following Whitfield and Baines (1990), the basic parameters that influence the behaviour of a turbomachine are collected in the form of a functional relation shown by equation (1).

$$\eta_{ts} = f\left(ER_{ts}, \frac{\dot{m} \sqrt{\frac{RT_t}{\gamma}}}{P_t \pi \frac{d^2}{4}}, \frac{\omega d \pi}{60 \sqrt{RT_t \gamma}}, Re, \gamma\right) \quad (1)$$

With the assumption of single working fluid passes through the turbine and fully developed turbulent flow regime, equation (1) is simplified into a more common form as shown by equations (2) and (3).

$$\eta_{ts} = f(\phi, \psi, Ma) \quad (2)$$

$$\eta_{ts} = f(N_s, d_s, Ma) \quad (3)$$

Where ϕ , ψ , N_s , d_s and are the flow and loading coefficients, specific speed and specific diameter and are defined by equations (4) to (7) respectively.

$$\phi = \frac{C_{m5}}{U_4} \quad (4)$$

$$\psi = \frac{\Delta h_{actual}}{U_4^2} \quad (5)$$

$$N_s = \frac{\omega \sqrt{Q_5}}{\Delta h_s^{0.75}} \quad (6)$$

$$d_s = \frac{d_4 \Delta h_s^{0.25}}{\sqrt{Q_5}} \quad (7)$$

Since the ORC turbines are often operating at high expansion ratios, the effect of compressibility (Mach number) is quite significant on the expander efficiency and should be included in the final correlation. However, the Mach number is always an outcome of the turbine design procedure and there is no prior knowledge of this parameter unless the detailed turbine design procedure is conducted. Since the aim of this study is to provide a correlation that can estimate the turbine efficiency without performing the turbine design process, which is clearly impractical, and using only non-dimensional parameters, volumetric expansion ratio (VR) introduced by Macchi and Perdichizzi (1981) is also included in the analysis to address the effect Mach number. VR is the ratio of the turbine stage outlet volumetric flow rate to the turbine stage inlet volumetric flow rate and is defined by equation (8).

$$VR = \frac{Q_5}{Q_1} \quad (8)$$

VR is a more meaningful parameter (in place of expansion ratio or Mach number) that explicitly correlates the degradation of turbine efficiency due to the high Mach numbers (Macchi and Perdichizzi, 1981). In other words, VR accounts for the compressibility effect in a more generalized way than other equivalent parameters (expansion ratio and Mach number) and one can estimate its value from the cycle analysis (based on the desired cycle requirements) and without the need for completing the turbine design procedure. Hence, the efficiency of organic turbines (characterized by high Mach numbers) can be more accurately explained in the following functional form:

$$\eta_{ts} = f(\varphi, \psi, VR) \quad (9)$$

In the classical work of Balje (1981) the turbine efficiency is correlated against N_s and d_s in which the best turbine designs lay on the Cordier line. The optimum radial turbine efficiencies are obtained at N_s values in the range of 0.5 to 0.7, however, the selection of an appropriate N_s does not necessarily yield the optimum turbine design. The selection of the optimum N_s can immediately achieve the optimum d_s and eventually the turbine rotor tip diameter, though, no other information is provided regarding to other turbine geometry. Chen and Baines (1994) employed data from a wide range of designs and various applications to correlate the radial turbine efficiencies against the other set of non-dimensional parameters as φ and ψ . They showed that the maximum radial turbine efficiency is obtained at the loading and flow coefficients in the range of 0.9 to 1 and 0.2 to 0.3 respectively. The use of φ - ψ couple is advantageous compared to N_s - d_s as more information about the turbine principal dimensions such as passage areas, inlet and exit blade height and velocity triangles is achieved. The N_s - d_s or φ - ψ performance correlation charts (such as Balje, 1981 and Chen and Baines, 1994) are often dated and may not accurately represent the performance of the modern radial turbines. In addition, atypical characteristics of ORC systems such as high expansion ratios (high Mach number), real gas behavior of working fluids and small turbine dimensions makes the use of these conventional charts questionable. Therefore, it is necessary to update these charts based on real gas properties and advanced loss models while employing an optimization scheme to maximize the radial turbine efficiency.

3. INTEGRATED MODELING AND OPTIMIZATION OF RADIAL TURBINE AND ORC SYSTEM

The methodology for the integrated modeling of the ORC system with the mean-line modeling of radial turbine is followed by the previous works of authors (Rahbar *et al.* 2015a, Rahbar *et al.* 2015b). The ORC system consists of four main components as the evaporator, turbine, condenser and pump. The turbine itself consists of the volute, nozzle and rotor. Figure 1 shows the schematic of the ORC and radial turbine together with their corresponding temperature-entropy and enthalpy-entropy diagrams respectively. The turbine-ORC model is developed in the EES software to utilize its reliable thermodynamic property functions for real gas modeling of the turbine-ORC system. EES uses the fundamental (Helmholtz free energy) equation of state for determining the thermodynamic properties of the selected organic fluids and details of which can be found in (Lemmon *et al.* 2006). Modelling of the radial turbine is based on a one-dimensional assumption in which there is a mean streamline through the stage that represents the average of passage condition and the thermodynamic properties and flow features (i.e. velocity triangles) are obtained at key stations (shown in Figure 1) across the mean line. With the choice of turbine input variables listed in Table 1 and the initial estimate of the turbine efficiency, the model determines the key geometry parameters for the rotor, nozzle and volute. Using the calculated geometry and the well-established loss correlations the model determines a more accurate estimation of turbine performance. This process is repeated until convergence is achieved for the turbine efficiency. Then with the calculated turbine exit thermodynamic properties (T_5 , P_5 , h_5 , S_5) and the ORC model input variables listed in Table 1, the main cycle parameters such as thermal efficiency, net power output and pump power consumption are determined. However, selection of arbitrary values from the range of input parameters shown in Table 1 does not necessarily yields the maximum efficiency for the turbine. Therefore an optimization scheme called genetic algorithm is coupled with the turbine-ORC model to maximize the radial turbine efficiency based on the input variables of Table 1. Moreover, the

optimization algorithm is constrained by some of the critical turbine geometry parameters, flow features and ORC characteristics to assure the feasibility of the optimized turbine dimensions and to achieve high cycle and turbine performances. Figure 2 shows the flow chart of the turbine-ORC model with integrated optimization scheme that details the overall procedure. More information regarding to the detailed modeling, optimization procedure and the imposed constraints can be found in (Rahbar *et al.* 2015a, Rahbar *et al.* 2015b). The turbine loss models employed in this study are expressed as total pressure loss coefficients and include the profile, incidence, blade loading, hub to shroud, tip clearance and distortion losses which are shown by equations (10) to (15) (Aungier, 2006). It should be mentioned that the majority of loss models are developed for the gas turbines with air or flue gases as the working fluids and due to the lack of experimental data in the open literature about the performance of ORC with small-scale organic radial turbines, such loss models should be taken with some care. However, the relative comparison of the performance with different fluids is believed to be primarily correct.

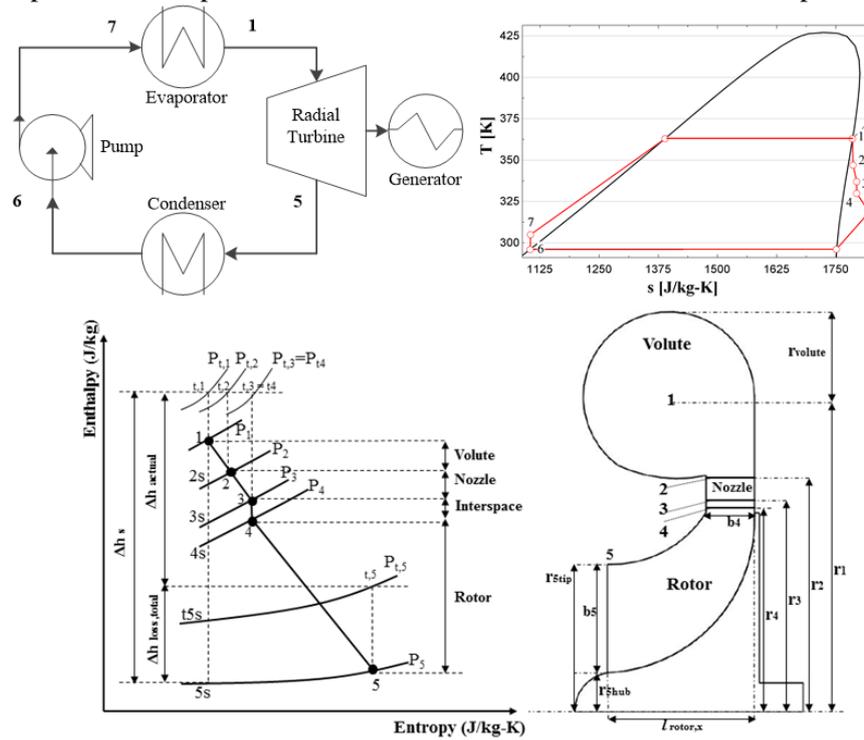


Figure 1: Schematic of the ORC system (top left), ORC temperature-entropy diagram (top right), radial turbine section view (bottom right), radial turbine enthalpy-entropy diagram (bottom left),

Table 1: Input parameters of the turbine-ORC model

Parameter	Unit	Value/Range
Turbine inlet total temperature ($T_{t,1}$)	K	343 - 393
Turbine inlet total pressure ($P_{t,1}$)	kPa	Saturation pressure
Turbine inlet degree of super heating ($T_{superheating}$)	K	1 - 5
Turbine total-to-static expansion ratio (ER_{ts})	-	2 - 8
Turbine rotational speed (ω)	rpm	30000 - 70000
Turbine loading coefficient (ψ)	-	0.7 - 1.4
Turbine flow coefficient (ϕ)	-	0.15 - 0.5
Turbine rotor exit absolute flow angle (α_5)	deg	-15 - 15
Turbine rotor exit hub to inlet radii ratio (r_{shub}/r_4)	-	0.2 - 0.3
Turbine nozzle inlet to exit radii ratio (r_2/r_3)	-	1.2 - 1.3
Turbine volute swirl coefficient (SC)	-	0.95
Turbine volute pressure loss coefficient (k_{volute})	-	0.1
ORC mass flow rate of working fluid (m)	kg/s	0.2 - 1
ORC pump efficiency (η_{pump})	-	0.7
ORC generator efficiency ($\eta_{generator}$)	-	0.96
ORC mechanical efficiency ($\eta_{mechanical}$)	-	0.96

$$Y_{profile} = \frac{2\Theta + \Delta^2}{(1 - \Delta)^2} \quad (10)$$

$$Y_{incidence} = \cos^2(\alpha_4 - \arctan(\frac{C_{m4}}{\sigma[U_4 - C_{m4} \tan(\beta_4)]})) (\frac{P_{rel,t,4} - P_4}{P_{rel,t,5} - P_5}) \quad (11)$$

$$Y_{blade,loading} = \frac{1}{24} \left[\frac{2\pi \frac{|r_5 C_{\theta 5} - r_4 C_{\theta 4}|}{l_{rotor,x} Z_{rotor}}}{W_5} \right]^2 \quad (12)$$

$$Y_{hub,to,shroud} = \frac{1}{6} \left[\frac{\kappa b_5 \frac{W_4 + W_5}{2}}{W_5 \cos(\alpha_5)} \right]^2 \quad (13)$$

$$Y_{tip,clearance} = \frac{0.816 \sqrt{2\rho_{average} (P_{rel,t,4} - P_4)} l_{rotor,x} Z_{rotor} \varepsilon}{\dot{m}} (\frac{P_{rel,t,4} - P_4}{P_{rel,t,5} - P_5}) \quad (14)$$

$$Y_{distortion} = \left(\frac{\frac{r_1 C_1}{r_2} - C_{\theta 2}}{C_2} \right)^2 \quad (15)$$

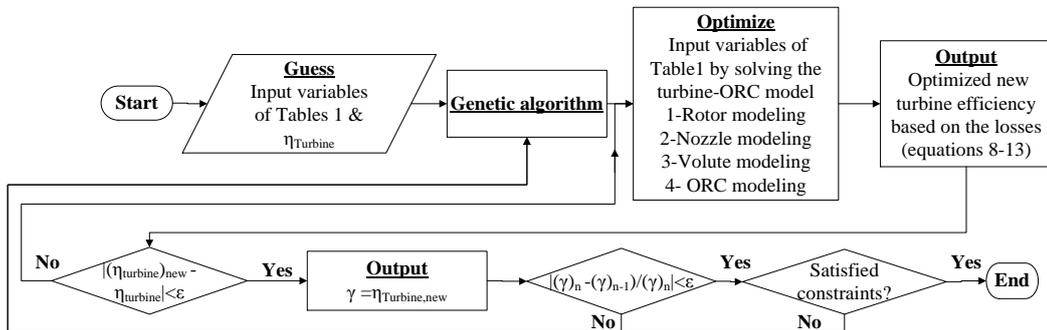


Figure 2: Flow chart for integrated modelling of turbine and ORC system with embedded optimization algorithm

4. RESULTS AND DISCUSSION

In order to conduct the optimization of turbine efficiency it is necessary to determine which parameters have the most significant effects on $\eta_{turbine}$. Following (Rahbar *et al.* 2015a, Rahbar *et al.* 2015b), among the listed parameters in Table 1, $T_{t,1}$, $P_{t,1}$, $T_{superheating}$, ER_{ts} , ω , ψ , φ , α_5 and m have the most significant effects on the $\eta_{turbine}$ and are included in the optimization using genetic algorithm. In order to correlate the variation of the optimized turbine efficiency with non-dimensional parameters and update the conventional φ - ψ and N_s - d_s charts the following specification for the design parameters are set to cover a wide range of designs.

$$\begin{aligned} T_{t,1} &\in (343 : 393 : 10) \\ \psi &\in (0.7 : 1.4 : 0.1) \\ \varphi &\in (0.15 : 0.5 : 0.05) \end{aligned} \quad (16)$$

Where the first and second terms are the upper and lower boundaries and third term is the step of variation for each parameter. The optimization is conducted for all possible combination of the above design parameters, however, only the optimized updated ϕ - ψ and N_s - d_s charts for the turbine inlet temperature of 373K are shown for brevity. As presented in Figure 3, the maximum turbine efficiencies are achieved at low flow coefficients with the maximum value of 86% obtained by R1233zd. In the ϕ - ψ performance charts, the maximum turbine efficiencies vary in the range of 82% to 86% compared to the maximum value of 88% shown by Chen and Baines (1994). This is due to the higher expansion ratios of ORC turbines (about 8) and corresponding supersonic losses compared to the conventional radial gas turbines with the maximum expansion ratios of about 4. In addition, implementation of back swept blading at rotor inlet is advantageous since the turbine efficiency increases at loading coefficients in excess of unity (Rahbar *et al.* 2015b). Comparing the ϕ - ψ charts shown in Figure 3 with Chen and Baines (1994) reveals that the turbine efficiency increases by a maximum of 3% at loading coefficients greater than unity. In other words the contours of maximum efficiency are slightly shifted to higher loading coefficients. Figure 4 presents the optimized updated N_s - d_s performance charts for the four investigated fluids. Similar to Balje (1981) diagram, the optimum region of N_s - d_s still exists in all charts in which the line of maximum efficiency is equivalent to the theoretical curve that almost gives $N_s d_s = 2$. But in contrast to the conventional radial turbines that the maximum efficiency occurs at the specific speed values of between 0.5 to 0.7, for ORC turbines this optimum value has been shifted to lower specific speed values of between 0.35 to 0.55 as shown in Figure 4.

Although the optimized performance charts shown in Figures 3 and 4 are essential for performance prediction of the ORC turbines, it is more beneficial to present the variation of the optimized turbine efficiency in the form of a mathematical equation. Therefore, linear regression analysis is employed in order to correlate the optimized turbine efficiencies with volumetric expansion ratio, flow and loading coefficients using all the generated design points. Figure 5 shows the linear regression plot for both the turbine inlet temperature of 373K (5a) and also for the complete range of inlet temperature from 343K to 393K (5b). As can be seen in Figure 5 the values of R^2 are quite high and assures that the regression analysis can fairly accurately predict the performance of the ORC radial turbines. Equation 17 shows the first order polynomial obtained based on this regression analysis using all the 417 created design points shown in Figure 5(b) and correlates the turbine efficiency with the volumetric expansion ratio, loading and flow coefficients.

$$\eta_{turbine} = 0.925 - 0.416\phi + 0.0279\psi - 0.00675VR \quad (17)$$

It should be underlined that the obtained correlation shown by equation 17 is assumed to be a preliminary correlation for performance prediction of ORC radial turbines and is considered as the initial step towards improvement in performance prediction of ORC based on the radial turbines. This is primary due to the fact that the employed loss models are originally developed for gas turbines using air as the working fluid and they should be used with some care. In addition, due to the lack of experimental data for performance of the ORC turbines it is required to validate the developed correlation with real test data upon their availability in open literature. However, it is believed that the proposed methodology is quite novel and has great potential to be further improved and extended for a wider range of organic fluids and broader range of operating conditions.

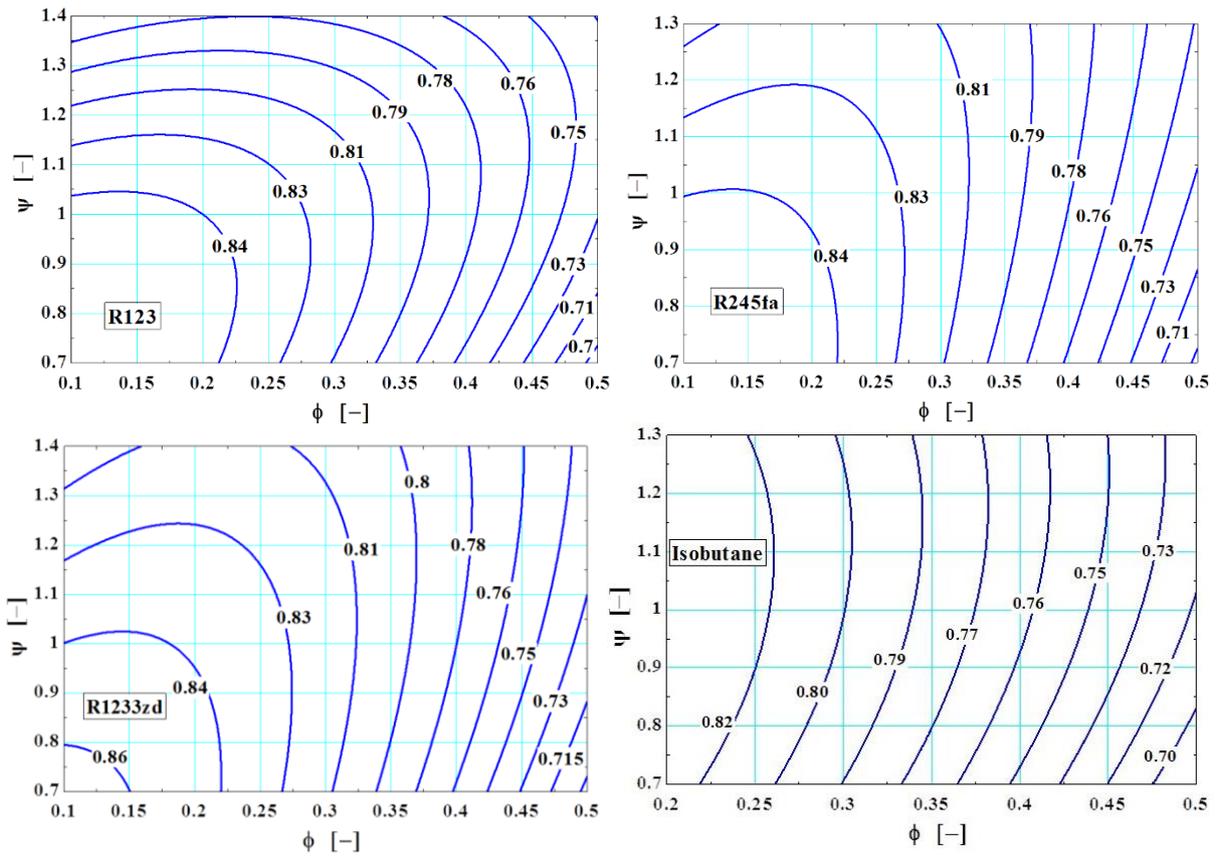


Figure 3: Contours of optimized turbine efficiency based on flow and loading coefficients for four organic fluids as R123, R245fa, R1233zd and isobutane at turbine inlet temperature of 373K

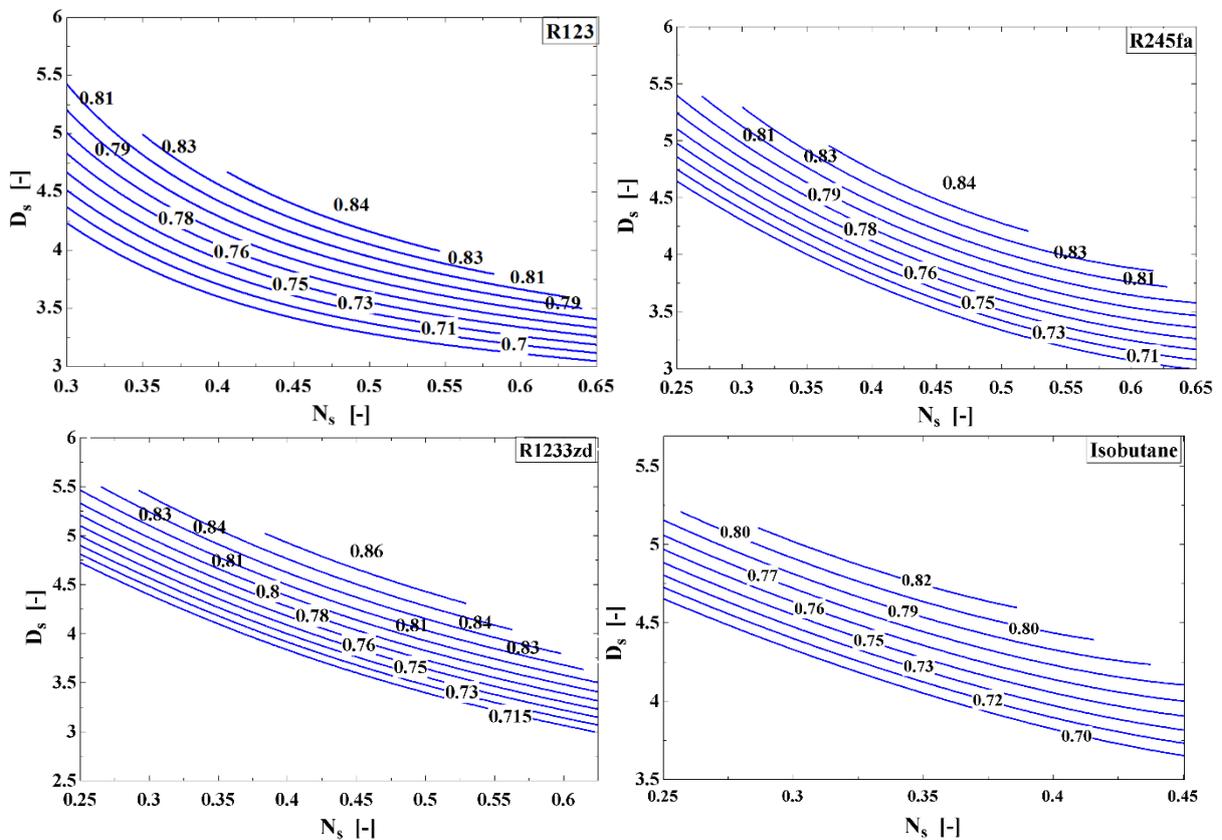


Figure 4: Contours of optimized turbine efficiency based on specific speed and specific diameter for four organic fluids as R123, R245fa, R1233zd and isobutane at turbine inlet temperature of 373K

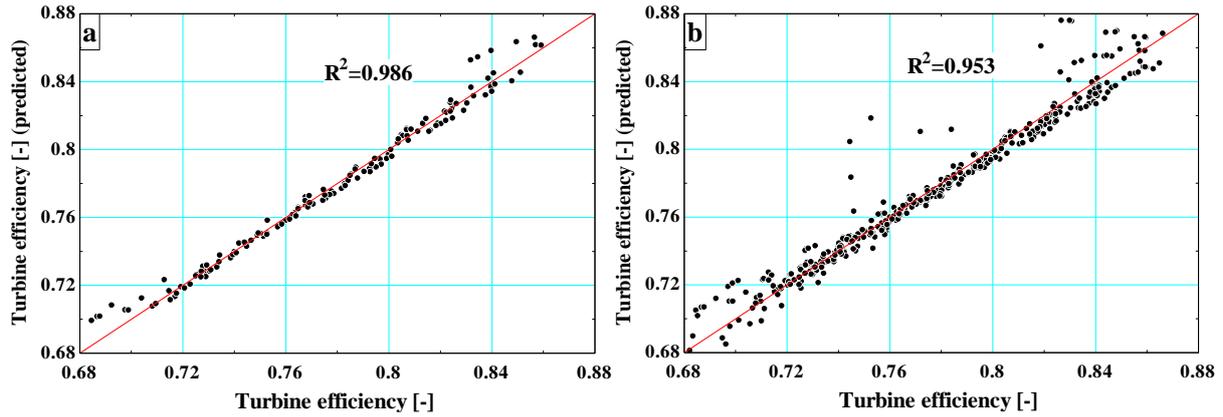


Figure 5: Predicted optimized turbine efficiency as a function of turbine efficiency at turbine inlet temperature of 373K (a) at the full range of turbine inlet temperature from 343K to 393K (b)

5. CONCLUSIONS

This study presents a novel approach that combines the modeling of the ORC system with modeling of radial turbine and allows for optimization of turbine efficiency based on a wide range of input variables. The paper shows that the conventional performance charts are no more sufficient for reliable performance predication of ORC radial turbines. These charts are updated and optimized based on the most advanced loss models and characteristics of organic fluids and ORC system (i.e. real gas behavior and high expansion ratios). The updated performance charts showed that there are considerable variations between them and the conventional ones as the contours of maximum efficiency are shifted to larger loading coefficients and smaller specific speeds respectively. In addition, the optimized charts showed that the maximum efficiency of ORC radial turbines are about 2% lower than the maximum efficiency of radial gas turbines. The R^2 values from the regression analysis showed that the established first order polynomial correlation can fairly accurately predict the performance of ORC turbines, though, it is required to be further modified and validated with the ORC experimental data (which are very scarce in the open literature). These new charts and the correlation could be a useful tool in general optimization procedures of the ORC systems that avoid any arbitrary assumption of turbine efficiency and yield a more accurate estimation of the turbine performance.

NOMENCLATURE

b	blade height	(m)
C	absolute velocity	(m/s)
d	diameter	(m)
d_s	specific diameter	(-)
ER	expansion ratio	(-)
h	enthalpy	(J/kg)
Δh_{actual}	actual specific enthalpy drop	(J/kg)
Δh_s	isentropic specific enthalpy drop	(J/kg)
l	flow path length	(m)
m	mass flow rate	(kg/s)
Ma	Mach number	(-)
N_s	specific speed	(-)
P	pressure	(Pa)
Q	volumetric flow rate	(m ³ /s)
R	gas constant	(J/kg-K)
r	radius	(m)
Re	Reynolds number	(-)
T	temperature	(K)
U	rotor blade velocity	(m/s)
VR	volumetric expansion ratio	(-)

W	relative velocity	(m/s)
Y	total pressure loss coefficient	(-)
Z_{rotor}	number of rotor blades	(-)
α	absolute flow angle to meridional	(degree)
β	relative flow angle to meridional	(degree)
κ	mean surface curvature	(m ⁻¹)
σ	slip factor	(-)
γ	specific heat ratio	(-)
ε	rotor casing clearance	(m)
η	efficiency	(-)
φ	flow coefficient	(-)
ψ	loading coefficient	(-)
ω	rotational velocity	(rad/s)
Θ	normalized momentum thickness	(-)
Δ	normalized mass thickness	(-)

Subscript

1-5	stations across the turbine
m	meridional direction
rel	relative value in rotating coordinate system
t	total conditions
ts	total to static
x	axial direction
θ	tangential direction

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