

ADVANCED THERMODYNAMIC MODEL OF ORGANIC RANKINE CYCLE

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ABSTRACT

Low-grade heat from geothermal or industrial processes is an eco-friendly resource for electric power production. The Organic Rankine Cycle (ORC) has become a popular means to exploit these energy sources. This growing popularity has resulted in the need for rapid, accurate simulation tools for plant design and specification.

In this paper, an advanced, steady state, thermodynamic model of an ORC is developed. The model is composed of several sub components including: an evaporator and condenser, centrifugal pump, turbine expander, and pipe elements. Each of these components is modeled independently and their inputs/outputs are combined together to form the overall system.

The model is developed using simple programming language (VBA) in excel and utilizes NIST Refprop for calculation of state properties. This medium was chosen because of its simplicity and low cost; however, the general model structure can easily be implemented in any programming language. The model predictions are validated against field data collected from ORC systems operating at several evaporating and condensing conditions.

1. INTRODUCTION

Rising energy costs, the demand for improved system efficiency, and government programs to reduce plant emissions have led to a surge in the popularity of the Organic Rankine Cycle (ORC.) Advanced modeling techniques are required to estimate the electrical power produced by an ORC system and to choose appropriate system components. The demand for simulation tools which can quickly and cheaply accomplish these tasks has grown in lockstep with ORC demand.

Commercially available process simulators (e.g. ASPEN HYSYS, CHEMCAD) are powerful but often expensive and cumbersome to work with. These “all encompassing” simulators are excellent for plant designs yet lack modeling for key components necessary to develop a high fidelity ORC system simulator. The following paper will describe the development of a low cost, flexible model written in Excel VBA. The model has been used to design over 100 plants and shows excellent agreement with field installations.

The model is divided into three main sections:

- 1) The working fluid loop
- 2) The evaporator
- 3) The condenser

The working fluid loop model is composed of several component models arranged in series to represent the ORC system. The following sections will describe each component model in detail.

2. MODELING METHODOLOGY

2.1 Pipe Section Model

The pipe model estimates the pressure and temperature losses experienced by the fluid as it flows through a pipe section. It links the major components (e.g. pump, evaporator, etc.) together in the working fluid loop model. The inputs are temperature, pressure, and mass flow and the outputs are temperature and pressure losses. Additionally there are several descriptive inputs: roughness, height, length, diameter, overall heat transfer coefficient, and ambient temperature. The pressure drop is found using the Darcy-Weisbach equation.

$$\Delta P = 8\pi f L \frac{\dot{m}^2}{\rho_{in,pipe} D^3} \quad (1)$$

The inlet density in Equation (1) is calculated from the inlet temperature and pressure. The Darcy friction factor in Equation (1) is found from an iterative solution to the equation below, Colebrook (1939).

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\frac{\epsilon}{3.7D} + \frac{2.51}{Re\sqrt{f}} \right) \quad (2)$$

The Reynolds number in Equation (2) is calculated using the inlet density and viscosity. These properties are calculated at the inlet temperature and pressure. Treating the pipe section as a cylinder with 1-D convective heat transfer and a constant ambient temperature, the temperature drop across the pipe section is calculated by (Incropera and DeWitt 2002):

$$\Delta T = (T_{amb} - T_{in}) e^{-\frac{\pi U D L}{\dot{m} c_p}} \quad (3)$$

The overall heat transfer coefficient in Equation (3) is adjusted based on test data of a production ORC. The specific heat is calculated at the pipe inlet temperature and pressure.

The pressure loss of system fittings within the working fluid loop (i.e. filters, valves, etc.) may be modeled as an equivalent length of pipe for a given diameter.

2.2 Pump Model

The pump model simulates the operation of a centrifugal pump as utilized in the Calnetix ORC product. The inputs are inlet temperature, inlet pressure, mass flow and outlet pressure. The outputs are the exit temperature and motor power. The calculation begins with determining the inlet density, enthalpy, and entropy at the inlet temperature and pressure. Next the volumetric flow and hydraulic head are calculated by:

$$G = \frac{\dot{m}}{\rho_{in}} \quad (4)$$

$$\Delta H = \frac{(P_{out} - P_{in})}{\rho_{in} g} \quad (5)$$

The values for volumetric flow rate and hydraulic head are input into a table map which outputs the pump efficiency, η_{pump} .

$$\eta_{pump} = MAP(\Delta H, G) \quad (6)$$

Next the ideal outlet enthalpy is calculated at the inlet entropy and outlet pressure. The true pump outlet enthalpy is calculated using:

$$h_{out} = \eta_{pump}(h_{out,s} - h_{in}) + h_{in} \quad (7)$$

Combining the output from Equation (7), the mass flow, and inlet enthalpy, the pump power is found using:

$$W_{pump} = \dot{m}(h_{out} - h_{in}) \quad (8)$$

Finally the outlet temperature may be calculated at the outlet pressure and outlet. The MAP used by Equation (6) was developed experimentally from the centrifugal pump model used in Calnetix ORC products. Maps for other types of pumps may be developed in the same manner.

2.3 Integrated Power Module (IPM)

The integrated power module, IPM, plays a pivotal role in the working fluid loop model by setting the system mass flow and determining the grid power output. The Calnetix IPM consists of a nozzle, turbine wheel, rotor, and generator stator. The rotor/wheel assembly is levitated by magnetic bearings within the generator stator, see Figure 1 below.

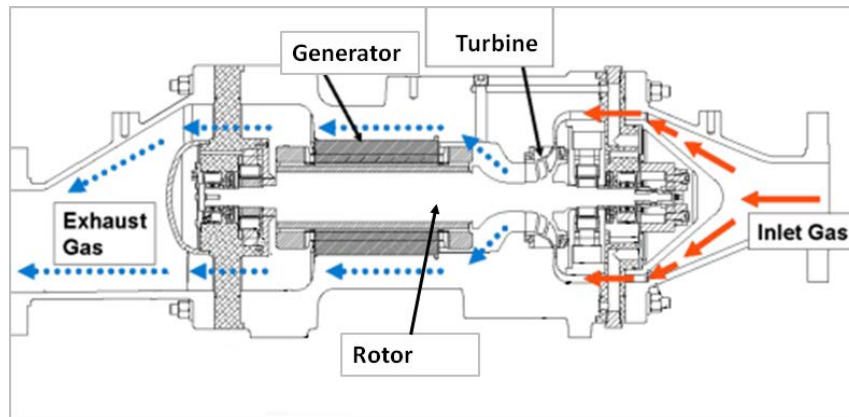


Figure 1: Calnetix IPM

The working fluid flows into the nozzle, through the turbine, over the rotor, and around the generator. Heat from turbine efficiency losses, rotor windage, and generator efficiency losses are added into the fluid stream increasing the exit enthalpy. These losses are estimated using table maps and correlations developed using CFD and empirical test data.

The inputs to the IPM model are inlet temperature, inlet pressure, outlet pressure, and turbine speed. Additionally there are several internal variables used from the previous calculation. These values are the static speed of sound at the nozzle inlet a^* , the static density at the nozzle inlet ρ^* , the dimensionless enthalpy drop across the wheel, the dimensionless wheel rotation speed, and the flow parameter at the wheel exit. The outputs are grid power, temperature at the IPM outlet, and working fluid loop mass flow.

The model calculation is carried out in several steps. Initially the internal input variables are input into map files which represent the turbine operation. The outputs are the isentropic efficiency (η_T) of the nozzle turbine system, the dimensionless inlet flow parameter (ϕ_{in}), and the turbine exit swirl angle (θ).

$$\eta_T = MAP \left(\frac{\Delta h_s}{(a^*)^2}, \frac{N}{a^*} \right) \quad (9)$$

$$\phi_{in} = MAP \left(\frac{\Delta h_s}{(a^*)^2}, \frac{N}{a^*} \right) \quad (10)$$

$$\theta = MAP \left(\frac{\Delta h_s}{(a^*)^2}, \frac{N}{a^*} \right) \quad (11)$$

Next the turbine exit swirl angle and wheel exit flow parameter are used to find the percentage pressure drop across the generator, $\Delta P_{Gen} \%$.

$$\Delta P_{Gen} \% = MAP(\theta, \phi_{out}) \quad (12)$$

The pressure at the wheel outlet is determined from:

$$P_{out} = (1 + \Delta P_{Gen} \%) P_{out,IPM} \quad (13)$$

Next the entropy and enthalpy at the nozzle inlet are calculated at the IPM inlet temperature and pressure. The ideal enthalpy at the wheel exit is calculated at the wheel outlet pressure and entropy at the nozzle inlet. The true enthalpy at the wheel exit is calculated by combining values from:

$$h_{out} = h_{in} - \eta_T (h_{in} - h_{out,s}) \quad (14)$$

Next the system mass flow is calculated from the inlet flow parameter (ϕ_{in}), static speed of sound at the nozzle inlet (a^*), and static density at the nozzle inlet (ρ^*). The model accounts for turbine leakage as follows:

$$\dot{m}_T = (1 - \alpha) \dot{m} \quad (15)$$

Combining the values from Equations (14), (15), and the nozzle inlet enthalpy, the turbine work may be calculated as:

$$W_T = \dot{m}_T (h_{in} - h_{out}) \quad (16)$$

The generator converts the turbine work calculated in Equation (16) to electrical power. A portion of the mechanical work is lost due to viscous effects acting on the rotor and wheel outer surfaces. These losses are called "windage" losses and are calculated through the aid of correlations from Norris (1970) and Vrancik (1968.) Additionally there are electrical losses due to resistance in the copper windings of the stator and iron losses due to changes in the magnetic field. An energy balance across the wheel/rotor control surface results in:

$$W_{Gen} = W_T - Q_{Loss} - W_{windage} \quad (17)$$

Where Q_{loss} is the heat lost from generator due to inefficiencies. Equation (17) may be simplified by introducing the generator efficiency, η_{Gen} , which relates the portion of turbine work lost due to generator efficiencies as:

$$\eta_{Gen} = \frac{W_T - Q_{Loss}}{W_T} \quad (18)$$

Thus combining Equations (17) and (18) then generator work may be expressed as (19):

$$W_{Gen} = \eta_{Gen} W_T - W_{windage} \quad (19)$$

The generator efficiency is found from a map lookup with inputs turbine work and generator frequency. The final step in the grid power calculation is to account for losses due to inefficiencies in the electrical power rectification and conversion process. These losses are accounted for with the power electronics efficiency number, η_{PE} , and related to the generator power and grid power by:

$$W_{PE} = \eta_{PE} W_{Gen} \quad (20)$$

To determine the temperature at the exit of the IPM, the enthalpy at the exit of the generator must be determined. This is found by forming an energy balance for the control surface surrounding the fluid flowing into and out of the generator:

$$\dot{m}(h_{out,Gen} - h_{in,Gen}) = Q_{Loss} + W_{windage} \quad (21)$$

The enthalpy at the generator inlet is found from a mass weighted average of the enthalpy exiting the turbine and leakage enthalpy. Using Equation (15) and values for the wheel outlet enthalpy and nozzle inlet enthalpy:

$$h_{in,Gen} = (1 - \alpha)h_{out} + \alpha h_{in} \quad (22)$$

Thus using Equations (18), (21), and (22) and solving for the generator outlet enthalpy, we arrive at:

$$h_{out,Gen} = (1 - \alpha)h_{out} + \alpha h_{in} + \frac{[(1 - \eta_{Gen})W_T + W_{windage}]}{\dot{m}} \quad (23)$$

Finally the exit temperature of the wheel is a function of the IPM outlet pressure and the enthalpy. The Calnetix ORC utilizes a reaction type turbine, thus the model is readily compatible with different types of reaction turbines. With appropriate map files, the model may be modified to accommodate other types of turbines.

2.4 Updating IPM Model Internal Variables

Once the calculations from section 2.3 are completed, the internal variables are updated for the next calculation loop. These values converge after several iterations. The first step in the update process is to calculate the static speed of sound and static density at the nozzle inlet.

The initial speed of sound is calculated at the nozzle inlet temperature and pressure. Using the value for enthalpy at the nozzle inlet found earlier, an intermediary enthalpy is found. by subtracting the square of the initial speed of sound:

$$h_i = h_{in} - \frac{a_i^2}{C} \quad (24)$$

Next the speed of sound is recalculated using the intermediary enthalpy, h_i , and the nozzle inlet entropy:

$$a_i = a(h_i, s_{in}) \quad (25)$$

The new speed of sound value is placed back into Equation (24) and the process is repeated until the value for a_i converges. This converged value is the static speed of sound at the nozzle inlet, $a_{in,nozzle}^*$. The final value of h_i is the static enthalpy at the nozzle inlet, h^* . The static density is now calculated from the value h^* and the entropy at the nozzle inlet.

Next the values for the static speed of sound at the nozzle inlet, nozzle inlet enthalpy, ideal nozzle exit enthalpy, and rotor speed are used to calculate the quantities $\Delta h_s / (a^*)^2$ and N/a^* . A new inlet flow parameter is calculated using the mass flow calculated by the IPM model. Finally the flow parameter at the wheel exit may be calculated using the procedure described above with the temperature, pressure, enthalpy, and entropy at the wheel exit.

2.5 Combining the Working fluid loop Components

Integrating the working fluid loop components is depicted graphically in Figure 2 below:

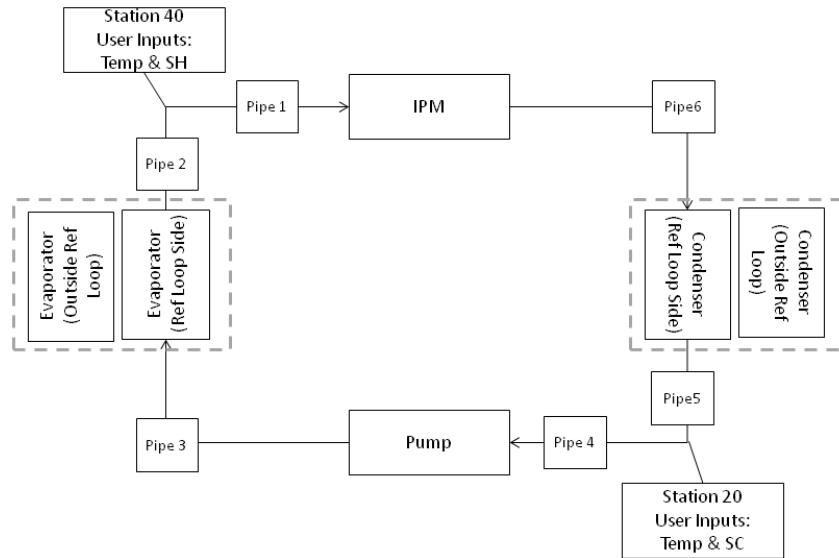


Figure 2: Working fluid loop Diagram

The overall user inputs to the working fluid loop model are temperature at station 40, superheat at station 40, temperature at station 20, and sub-cooling at station 20. From these values the pressure is calculated at stations 40 and 20 respectively. The pressure drop across each pipe element is added or subtracted from the P_{40} and P_{20} . Thus the pressure inputs to each component are calculated by:

$$P_{in,IPM} = P_{40} - \Delta P_1 \quad (26)$$

$$P_{out,IPM} = P_{20} + \Delta P_5 + \Delta P_{Cond} + \Delta P_6 \quad (27)$$

$$P_{in,Pump} = P_{20} - \Delta P_4 \quad (28)$$

$$P_{out,Pump} = P_{40} + \Delta P_2 + \Delta P_{Evap} + \Delta P_3 \quad (29)$$

The inlet temperature to each component is found using the temperature loss for each pipe section with Equation (3). In some cases the temperature loss is calculated in reverse and added to the user input temperature. The input/output temperature values are updated with each successive calculation of the working fluid loop. Convergence is achieved after several iterations.

2.6 Evaporator and Condenser Models

The evaporator and condenser models estimate the source mass flow and heat exchanger surface area needed. The user inputs are inlet temperature and pressure of the source, source side pressure loss coefficient, working fluid side pressure loss coefficient, exit temperature of the source, and overall heat transfer coefficient. Additionally these models take inputs of working fluid inlet temperature/pressure, outlet temperature/pressure, and heat exchanger duty which are all calculated by the working fluid loop model. The pressure drop on the source and working fluid sides are calculated by the minor loss equation:

$$P_{out} = P_{in} - \xi \frac{\dot{m}^2}{\rho_{in}} \quad (30)$$

The inlet density in Equation (30) is calculated at the inlet temperature and pressure. The pressure loss coefficient, ξ , is typically given by the heat exchanger manufacturer. The heat exchanger duty is calculated from the inlet enthalpy, outlet enthalpy, and mass flow on the working fluid side by:

$$Q = \dot{m}(h_{in} - h_{out}) \quad (31)$$

The source mass flow is calculated by dividing the heat exchanger duty calculated by Equation (31) by the enthalpy change between the inlet and outlet of the source:

$$\dot{m}_{source} = \frac{Q_{HEX}}{(h_{out,Source} - h_{in,Source})} \quad (32)$$

The inlet enthalpy for the source is calculated at the source inlet temperature and pressure. The outlet enthalpy is calculated from the source outlet temperature and pressure. The source outlet pressure is updated iteratively using Equation (30), (31), and (32).

The heat exchanger surface area is found using a discrete integration process. The process begins by determining the enthalpy and pressure step sizes for both the source side and working fluid sides of the evaporator and condensing with:

$$\Delta h = \frac{h_{out} - h_{in}}{k} \quad (33)$$

$$\Delta P = \frac{P_{out} - P_{in}}{k} \quad (34)$$

From Equations (33) and (34), a discretized pressure and enthalpy field for the source and working fluid sides is defined by:

$$h_i \in \{h_{in} + i\Delta h\} \text{ for } i = 0,1,2, \dots, k \quad (35)$$

$$P_i \in \{P_{in} + i\Delta P\} \text{ for } i = 0,1,2, \dots, k \quad (36)$$

By Equation (36)(37), the pressure field is evenly spaced with the enthalpy change for both the source and working fluid sides. The discrete temperature field may be calculated from each enthalpy and pressure pair as:

$$T_i \in \{T(h_i, P_i)\} \text{ for } i = 0,1,2, \dots, k \quad (37)$$

The heat flux through each heat exchanger section is defined for both the source and working fluid sides as:

$$Q_i \in \{(1/\dot{m})(h_i - h_{i-1})\} \text{ for } i = 1,2, \dots, k \quad (38)$$

Next the log mean temperature difference for each section is found from:

$$T_{i,LMD} \in \left\{ \frac{(T_{i,Source} - T_{k-i,Ref}) - (T_{i-1,Source} - T_{k-i-1,Ref})}{\ln \frac{(T_{i,Source} - T_{k-i,Ref})}{(T_{i-1,Source} - T_{k-i-1,Ref})}} \right\} \quad (39)$$

The index "i" in Equation (39) takes integer values from 0 to k. Using Equations (38) and (39) and the relationship between heat flux, the overall heat transfer coefficient, heat exchanger surface area, and log mean temperature difference:

$$(UA)_i \in \{Q_i/T_{i,LMD}\} \text{ for } i = 1,2, \dots, k \quad (40)$$

Finally the heat exchanger surface area is found by summing the values in Equation (40) and dividing by the user input overall heat transfer coefficient, U .

$$A = \frac{1}{U} \sum_{i=1}^k (UA)_i \quad (41)$$

The model described above is intentionally simple for the following reasons:

- 1) The model is iterated many times to arrive at the desired set point. A simpler component model reduces the overall computation time.
- 2) Complex models require parameters such as local friction factor which need to be determined experimentally or analytically across a broad range of fluid conditions. This data is often unavailable from heat exchanger manufacturers.

3. CASE STUDIES

The model described above is used extensively by Calnetix to assess new applications as well as validate unit performance at specific operating conditions. The data tabulated below is from 4 distinct applications in the field. The applications are all commercial, utilizing generated power within the facility or exporting power to local utilities. The working fluid in these cases was R245fa refrigerant. The model output in general shows good agreement with field data. The Gross Efficiency is defined as the gross electrical power exported to the grid divided by the rate of heat is transferred to the working fluid.

Table 1: Saturated Steam Plant- Japan

	Model	Field data
Heat Source Temp (°C)	145	145
Heat Source Pressure (Bar)	1.95	1.8
Heat Source Flow Rate (kg/s)	0.35	0.35
Coolant Temperature (°C)	26	26
Coolant Pressure (Bar)	1.38	No Value
Turbine Inlet Temperature (°C)	122	119
Turbine Inlet Pressure (Bar)	16.4	15.5
Condenser Exit Temperature (°C)	29	29.5
Condenser Exit Pressure (Bar)	1.9	1.95
Gross Power Output (kW)	100.7	101
Gross Efficiency (%)	11.1	11.0

Table 2: Hot Water from an Incineration Plant- Japan

	Model	Field data
Heat Source Temp (°C)	98.7	98.7
Heat Source Pressure (Bar)	2.9	2.9
Heat Source Flow Rate (kg/s)	7.25	12.28
Coolant Temperature (°C)	20.2	20.2
Coolant Pressure (Bar)	1.4	1.4
Turbine Inlet Temperature (°C)	67.8	67.8
Turbine Inlet Pressure (Bar)	4.9	4.9
Condenser Exit Temperature (°C)	21	21
Condenser Exit Pressure (Bar)	1.47	1.47
Gross Power Output (kW)	100.8	124.7

Gross Efficiency (%)	7.0	6.7
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Table 3: Geothermal Low Pressure Steam- Japan

	Model	Field data
Heat Source Temp (°C)	125	128
Heat Source Pressure (Bar)	2.1	2.9
Heat Source Flow Rate (kg/s)	0.47	0.45
Coolant Temperature (°C)	25	24
Coolant Pressure (Bar)	1.3	1.5
Turbine Inlet Temperature (°C)	120	126
Turbine Inlet Pressure (Bar)	17.3	18.2
Condenser Exit Temperature (°C)	31.4	29.9
Condenser Exit Pressure (Bar)	2.06	1.99
Gross Power Output (kW)	125	125
Gross Efficiency (%)	12	12

Table 4: Flue Gas from Clay Factory Using Intermediate Oil Loop- Europe

	Model	Field data
Heat Source Temp (°C)	148	160
Heat Source Pressure (Bar)	7	No Value
Heat Source Flow Rate (kg/s)	20.8	20
Coolant Temperature (°C)	19	28
Coolant Pressure (Bar)	1	1
Turbine Inlet Temperature (°C)	128	130
Turbine Inlet Pressure (Bar)	18.7	18.6
Condenser Exit Temperature (°C)	25.7	30
Condenser Exit Pressure (Bar)	1.8	2.8
Gross Power Output (kW)	125	125
Gross Efficiency (%)	12.6	12.4

4. CONCLUSION

The development of an advanced thermodynamic model of an Organic Rankine Cycle was discussed. The model is implemented in Excel VBA script and uses Refprop as an equation of state. Each major component of the ORC is modeled in detail and combined together to form the completed system. The model obviates the need for complex and expensive process simulation tools, and delivers a highly accurate output. This affords a high degree of confidence to Calnetix and its customers when selecting balance of plant components and arriving at sensitive capital investment decisions.

NOMENCLATURE

α	turbine seal leakage	(%)
ϵ	roughness height	(m)
θ	swirl angle	(°)
η	efficiency	(%)
ξ	minor loss coefficient	(MPa-m ³ -s ² /kg ³)
ρ	density	(kg/m ³)
Φ	flow parameter	(m ²)
A	surface area	(m ²)
a	speed of sound	(m/s)
C	unit conversion constant	(J-s ² /kg-m ²)
c_p	specific heat	(J/kg-°C)

D	diameter	(m)
f	Darcy friction factor	(-)
G	volumetric flow rate	(m ³ /s)
g	gravitational constant	(m/s ²)
H	hydraulic head	(m)
h	specific enthalpy	(J/kg)
k	number of heat exchanger sections	(-)
L	length	(m)
MAP	value lookup table	(-)
\dot{m}	mass flow rate	(kg/s)
N	rotational speed	(RPM)
P	pressure	(MPa)
Q	heat	(Watts)
Re	Reynolds number	(-)
T	temperature	(°C)
U	overall heat transfer coefficient	(W/m ² -°C)
W	work	(Watts)

Subscript

*	static condition
amb	ambient
Cond	condenser
Evap	evaporator
Gen	generator
i	index integer
in	inlet condition
IPM	integrated power module
LMD	log mean difference
Loss	heat loss
out	outlet condition
PE	condition or item related to the power electronics
pipe	condition or item related to the pipe model
Ref	working fluid side of the condenser or evaporator
s	isentropic
Source	source side of the condenser or evaporator
T	condition or item related to the turbine
Windage	energy loss associated with windage

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