EFFICIENCY CORRELATIONS FOR AXIAL FLOW TURBINES WORKING WITH NON-CONVENTIONAL FLUIDS

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ABSTRACT

This work aims at defining a set of general correlations for the estimation of axial-flow turbine efficiency in Organic Rankine Cycle (ORC) field. A dedicated numerical tool is used for the optimization of several hundreds of turbines and the results are presented in specific parameters (SP, V_r and Ns) according to similarity rules. The analysis is carried out for single, two and three stages turbines. For each case a correlation of efficiency at optimal rotational speed is calibrated in function of the equivalent single stage SP and the total isentropic V_r. Three sensitivity analyses are proposed in order to highlight the effects of each single parameter on stage efficiency. Finally, the effect of fluid choice on turbine performance and dimension is discussed with a numerical example.

1. INTRODUCTION

The energy market is today more and more oriented to technical solutions able to exploit renewable energy sources and waste heat from industrial processes with the aim at reducing air and water pollution and increasing systems efficiency. In this context Organic Rankine Cycle (ORC) is one of the most reliable and mature solution for the exploitation of various energy sources characterized by a small available thermal power and/or a low maximum temperature. Typical fields of application of ORC technology are the geothermal energy, the solar energy, the biomass combustion and the heat recovery from industrial processes. ORCs have a simple layout characterized by a limited number of components; in addition they are extremely flexible showing good off-design performances in a large range of working conditions. The main advantage of ORCs is the possibility to select the most appropriate working fluid among a wide list of candidates guaranteeing high efficiency cycles in a large range of applications (Astolfi *et al*, 2014a). Despite their simplicity, the study of ORCs requires cross-functional expertise because the design and the optimization of these plants is largely influenced by the availability of (i) accurate equations of state (EoS), (ii) reliable cost correlations and (iii) efficiency correlations able to describe components performance. In particular, the expander is the key component of an ORC and the estimation of its efficiency and dimension is crucial to obtain a reliable evaluation of system performance and cost. The assumption of a fixed expander efficiency, independent of (i) the operative conditions (inlet and outlet thermodynamic states), (ii) the working fluid and (iii) the expected power output, may lead to unrealistic results (Astolfi *et al*, 2014b). In ORC field different types of expanders are used depending on the plant power output and the expansion ratio. Axial flow turbine is the most common choice and it is used by main market leaders like Ormat and Turboden, while Exergy has recently introduced high efficiency radial outflow turbines, especially suitable for high volume ratio expansions (Xodo *et al*, 2013). Radial inflow turbines are designed by Atlas Copco and by Calnetix GE in a large range of dimensions: from few tens of kW to tens of MW in geothermal applications. Finally, positive displacement expanders (scroll, pistons or screw devices) are proposed for small size applications like solar power plants and automotive ORC. This work is focused on axial-flow turbines because of their large market share and their capability to cover a large range of applications with high isentropic efficiency.

Various correlations, charts and diagrams for the estimation of axial turbine efficiency are proposed in literature but usually they lack in to account some variables of crucial interest in ORC field. The use of complex and heavy fluids entails a design of ORC turbines which is different from either gas or steam turbines. Organic fluids generally operate at moderate temperatures and show a small isentropic enthalpy drop in expansion leading to the design of compact turbines with a reduced number of stages and low load coefficients (k_{is}). Peripheral speed is generally not a critical issue while in gas and steam turbines this is the key limiting factor in the selection of stage number due to both mechanical stresses and high temperatures. On the other hand, organic fluids usually have large volume ratios per stage and a low speed of sound. Both these aspects make the design of an ORC turbine a quite challenging task which cannot be faced without the support of a specific optimization software. Simple correlations such as Smith (1965) diagrams and Baljé and Binsley (1968) plots cannot catch these ORC turbines peculiarities. As already demonstrated by Macchi (1977), Macchi and Perdichizzi (1981) and Lozza *et al*. (1981), a rigorous design of an axial flow turbine should take into account real blade dimensions and the effects of Mach numbers especially if supersonic flows are present. These aspects are crucial for organic fluids, where blades are characterized by a large height variation, large flaring angles and supersonic velocities. In particular in Macchi (1977), the effect of both the volume ratio and the specific speed is analyzed for different single stage turbines operating with heavy and complex fluids while in Lozza *et al*. (1981) a sensitivity analysis is carried out varying the number of stages for turbines with two different size parameters. A further step ahead is provided in the work of Macchi and Perdichizzi (1981) with the definition of a map of efficiency for single stage turbines at optimized rotational speed as function of size parameter and volume ratio. The purpose of this paper is to propose an efficiency correlation for single, two and three stage axial flow turbines at optimized rotational speed in a wide range of volume ratios and dimensions.

2. SCOPE OF WORK AND METHODOLOGY

Except for some biomass applications, in ORC field each turbine is designed ex novo because each plant differs in the available thermal power and in the temperature of both the heat source and the cooling medium. Turbines in ORC can operate with different fluids in a large range of pressure and volume ratios, size and power output. For these reasons, it is interesting to adopt a non-dimensional approach which allows comparing optimal turbine designs on the basis of specific parameters. According to similarity rules (Dixon, 1998), the results achieved for a certain turbine stage can be extended to any other case if the stages respect the following conditions (Macchi and Perdichizzi, 1981):

1. They have the same specific speed;
2. The geometric similarity is fully verified (all the geometrical ratios are equal);
3. The flow is fully turbulent so that the Reynold number effects are negligible
4. the Mach numbers are similar;
5. The volumetric behaviour of the two fluids is the same, namely the volume flow rate variation across the stage is equal for the two fluids. This condition is verified if the two fluids are incompressible or if they are ideal gas with the same pressure ratio and the same heat capacity ratio;

ORC turbines ranges from micro scale to very big machines and so the geometrical similarity cannot be always verified because of the presence of technological constraints like the minimum trailing edge thickness and the minimum tip clearance gap. Very small turbines are intrinsically less efficient because of the increase of profile and leakage losses. On the other hand, high value of volume ratios affect the turbine stage design which requires converging diverging blades and relevant variation of blade height in a single row with detrimental effects on stage efficiency.

The parameters suggested as independent variables for a parametric analysis are: the size parameter $\left(SP = \frac{\mathrm{V}_{out,is}{}^{0.5}}{\Delta h_{is}{}^{0.25}}\right)$ and the volume ratio $\left(V_r = \frac{\mathrm{V}_{out,is}}{\mathrm{V}_{in}}\right)$ while the specific speed $\left(Ns = \frac{RPM}{60}\frac{\mathrm{V}_{out,is}{}^{0.5}}{\Delta h_{is}{}^{0.75}}\right)$ will be optimized for each case. The physical significance of SP, V_r and Ns and their influence on the turbine efficiency will be discussed later.

In order to obtain efficiency maps with a sufficient detail and covering a large range of ORC applications, hundreds of turbines have been optimized. Independent variables range between a minimum and a maximum value which is 0.02 m -1 m for SP and 1.2-200 for Vr.
The optimizations presented in next sections are realized assuming a complex ideal gas with a γ value equal to 1.05 as representative of a generic organic fluid. We found this simplifying hypothesis appropriate for the scope of the present analysis, i.e. to obtain quite accurate preliminary prediction of turbine efficiency for a large variety of ORC cycles and working fluids. The results in section 6 confirm the validity of this assumption. Of course, in the final turbine design a proper EoS should be considered.
In this study, Reynolds numbers are not considered as independent variables since for Re greater than 10^6 the effect on stage performance is negligible, while the influence of Mach numbers on blade geometry, flow angles and losses are accounted for. Even if the Mach numbers resulting from the adopted ideal gas assumption differ from the real ones, these deviations cause minor effects on the predicted stage efficiency (Macchi and Perdichizzi, 1981).
All the results presented in the next sections are obtained with Axtur tool, an *in house* optimization code for axial flow and radial outflow turbines developed by Macchi and Lozza at the Energy Department of the Politecnico di Milano. The code is based on a pseudo 1D approach and both blade channels geometry and velocity triangles are defined at mean diameter for each blade row (see Fig 1 for nomenclature details). Blade heights are hence obtained from continuity equation and the actual blade geometry is considered in efficiency losses calculation. Efficiency loss for each row is computed using correlations from literature to take into account the presence of the boundary layer, supersonic flows, flow angle variations and other losses.
Axtur, starting from a feasible initial point, performs multivariable constrained optimizations of turbines with a maximum stage number equal to three. Every stage is fully defined by nine parameters. Three of them define stage quantities: the stage isentropic load (k_{is}), the isentropic degree of reaction (r^*) and the isentropic volume ratio (V_r). The other six (three for each row) parameters are representative of stage geometrical ratios (o/s, o/b and b/r_m). These parameters are the optimization variables of the problem and they can be varied by the optimization algorithm between a lower and an upper bound whose values are defined according to the limits of the correlations used to compute the efficiency losses.
In addition, nonlinear constraints are considered for other variables of interest, like maximum Mach numbers, maximum number of blades and flaring angles and for technological limitations. Penalty factors are introduced if the upper or the lower bounds are not respected.

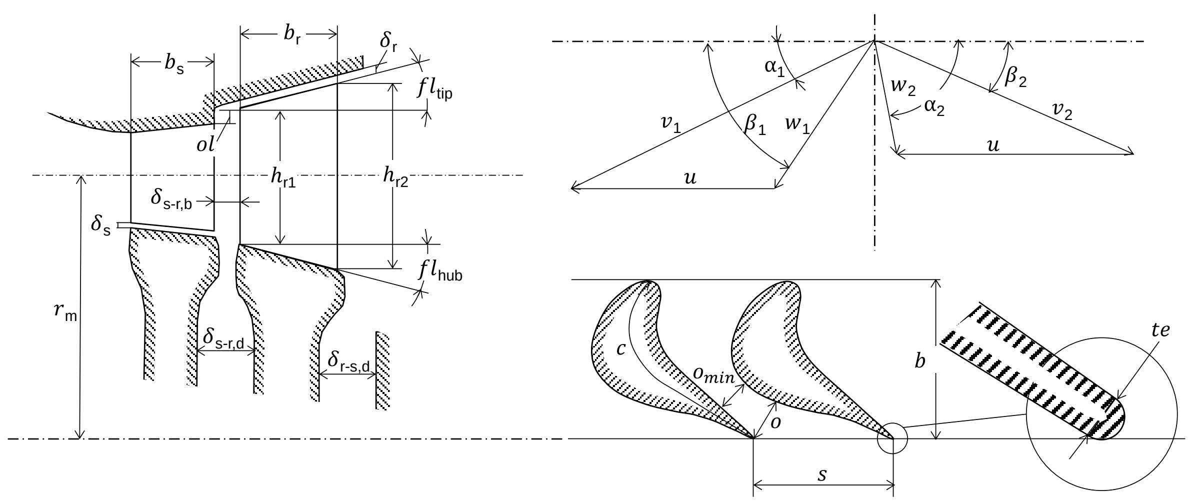

Figure 1 - Notation used in Axtur for the blade geometry and the velocity triangles. Converging-diverging blades are used by the code when Ma>1.4 otherwise $o = o_{min}$. In the velocity triangle the v vector represents the absolute velocity while w one the velocity relative to the rotor blade; u vector is the peripheral speed. Subscripts 1 and 2 refer to the inlet and the outlet of the rotor blade respectively.

For a single stage Axtur optimizes the total to static efficiency corrected by the fraction of kinetic energy recovered by the diffuser according to eq. 4.

$$\eta = \frac{\mathrm{W}}{\Delta h_{T-S} - \varphi_{\mathrm{E}} \frac{\mathrm{v}_{2,\mathrm{a}}^2}{2}} \quad (1)$$

Where φ_{E} is the efficiency of the diffuser; it is assumed that 50% of the kinetic energy of the discharge absolute velocity axial component can be recovered.

The approach is not completely rigorous because the recovery of kinetic energy entails a reduction of the pressure at turbine discharge and so a higher pressure drop and a higher power production while here the effect is accounted subtracting the same term from the denominator. As proved by Macchi [6], the approximation is generally valid and does not affect the quality of the solution in terms of turbine efficiency. For multistage turbine the code maximizes the power output imposing $\varphi_{\mathrm{E}} = 1$ for all the stages except the last one, but introducing annulus losses between blade rows.

The results provided by Axtur consist in a complete characterization of blade geometry in both blade to blade and meridional planes and velocity triangles. Furthermore, the breakup of the efficiency losses is reported considering the following effects:

- *Profile loss* (Craig&Cox [10]): due to the blade shape and effects related to friction, fluid vane deflection and boundary layer dissipation. This loss mainly depends on blade pitch, blade axial chord length, trailing edge thickness and roughness of blade surface, angular deflection and relative velocity ratio.
- *Secondary losses* (Craig and Cox, 1970): caused by secondary flow structures mainly described by passage vortex, horseshoe vortex, trailing edge vortex and corner vortex. These losses are affected by the same parameters which have influence on profile losses plus blade height.
- *Annulus losses* (Craig and Cox, 1970 or Kacker and Okapuu, 1981): due to the passage of fluid in the gap between two blade rows. This loss is calculated for all the stages except for the last one.
- *Leakage losses* (Craig and Cox, 1970): caused by the unwanted passage of fluid above blade tip whose expansion does not contribute to power production. This kind of loss is mainly related to radial clearance, blade length and blade overlap. They can be null for the first stator blades only.
- *Disk windage losses*: due to velocity gradient in the clearance between stator and rotor disk walls. They are strongly affected by rotational speed and by both absolute disk clearance and disk diameter.
- *Kinetic energy loss*: it is the fraction of kinetic energy of discharge velocity which cannot be recovered with the diffuser. Usually it is defined with a coefficient φ_e smaller than unit respect to the kinetic energy of the axial component.

Exit flow angles are computed according with the flow condition: (i) Subsonic flow (Ainley and Mathieson, 1951), (ii) Supersonic flow with after expansion (Vavra, 1969), (iii) Supersonic flow with converging diverging nozzle (Deich, 1965). Experimental data from Deich (1965) are used for accounting of additional losses related to supersonic flows at blade exit, while an empirical correlation introduced by Macchi (1977) accounts for losses related to relative transonic velocities at rotor inlet.

3. SINGLE STAGE TURBINES

More than five hundreds of turbines are optimized varying SP and V_r and optimizing Ns. It is important to remember that, according to similarity rules, the results here obtained are representative of any other turbine stage with the same set of independent parameters. Different volumetric fluid behavior and molar mass affect variables like speed of revolution, pressure ratio, temperature and enthalpy drop and mass flow rate but they have small influence on the final optimal design which has the same geometrical aspect and the same isentropic efficiency. The validity of this assumption is verified in the test case presented in chapter 6.

In Fig 2.a each point represents the maximum efficiency attainable for any combination of SP, Vr and Ns. It is possible to notice that for each couple of SP and V_r parameters an optimal specific speed is found, while detrimental effects on the efficiency can be highlighted increasing the V_r and decreasing the SP. As a general consideration, there is a large range in SP and V_r where the optimal specific speed is between 0.1 and 0.15, as already pointed out in Macchi [6]. For small SP and for high V_r the optimal Ns decreases down to 0.05. This is justified by the presence of lower and upper bounds on some geometrical dimensions like minimum blade height and maximum h/D ratio at the discharge section. In these cases, a Ns value above 0.1 entails an almost unfeasible design of the blade with a strong increase of secondary losses due to fluid leakages and high flaring angles.
The maximum point for every curve is representative of the maximum attainable efficiency at optimized rotational speed and optimal results are collected in a contour map reported in Fig 2.b. For a better understanding of the effects occurring in the definition of optimized stage geometry, three parametric analyses are proposed in the following. The first focuses on the effect of Ns at fixed SP and V_r, while the other two are carried out varying one by one V_r and SP at optimized rotational speed.

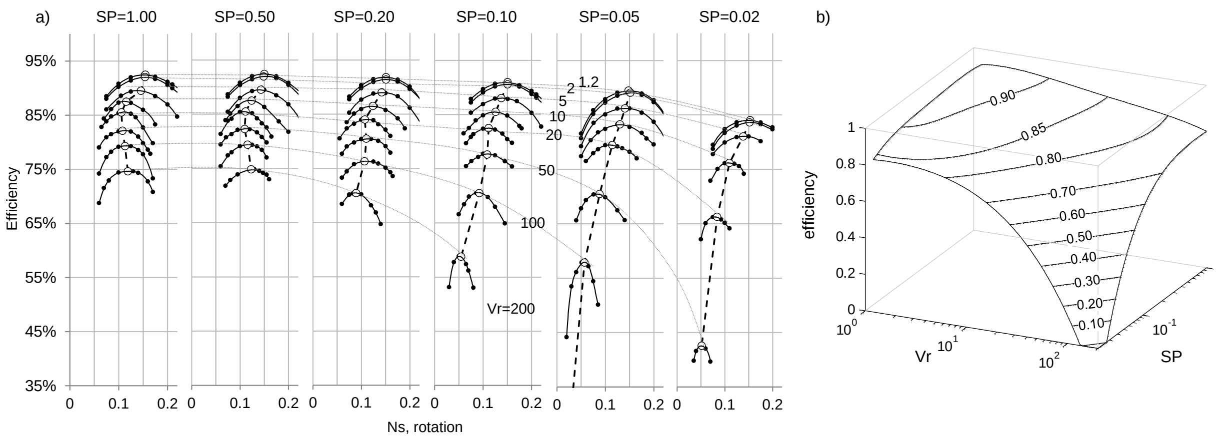

Figure 2 – a) Results for the single stage turbines. Black markers (•) are representative of the optimal turbine configuration for each combination of SP, Vr and Ns. White markers (○) identify the turbine designs at optimized rotational speed. b) map of efficiency for a single stage turbine at optimal Ns.

2.1. Effect of Ns

The first analysis regards the optimization of Ns for a single stage turbine at fixed SP and V_r equal to 0.4 m and 4 respectively (similar results are obtained for any other SP and Vr.combination) Results are reported in Fig 3. In this case, both the volume flow rate at turbine exit and the isentropic enthalpy drop are constant and so Ns parameter has a direct effect on the speed of revolution and turbine mean diameter. At low specific speed the turbine stages have large mean diameter because of the necessity to maintain optimal u velocity above a certain value, to reduce the stage load.
Stages in this region have small h/D parameters and they are affected by high leakage and secondary losses. Velocity triangles are representative of impulse stages with an almost axial absolute velocity v_2, thus the kinetic losses are minimized. Disk windage loss is noticeable because of the large diameter interested by this dissipation effect.
Increasing the rotational speed allows reducing both the secondary and leakage losses thanks to higher blades and a larger h/D ratio while the profile and the kinetic losses become more and more relevant. Optimal Ns value is equal to about 0.15 corresponding to well-proportioned turbine stages and an almost 50% reaction velocity triangle with an axial absolute velocity at turbine outlet.
On the other hand, for higher Ns values, the rotational speed is higher and the stage mean diameter is strongly reduced leading to high values of h/D parameter. Both the secondary and the leakage losses are

minimized but the distorted shape of the velocity triangles entails high values of discharge velocity which cannot be maintained in axial direction. As a consequence, kinetic energy losses increase because of the high value of v_2 and the presence of a tangential component which cannot be recovered by the diffuser. The trade-off between these opposite effects leads to the presence of an optimum value of Ns which yields the maximum efficiency.

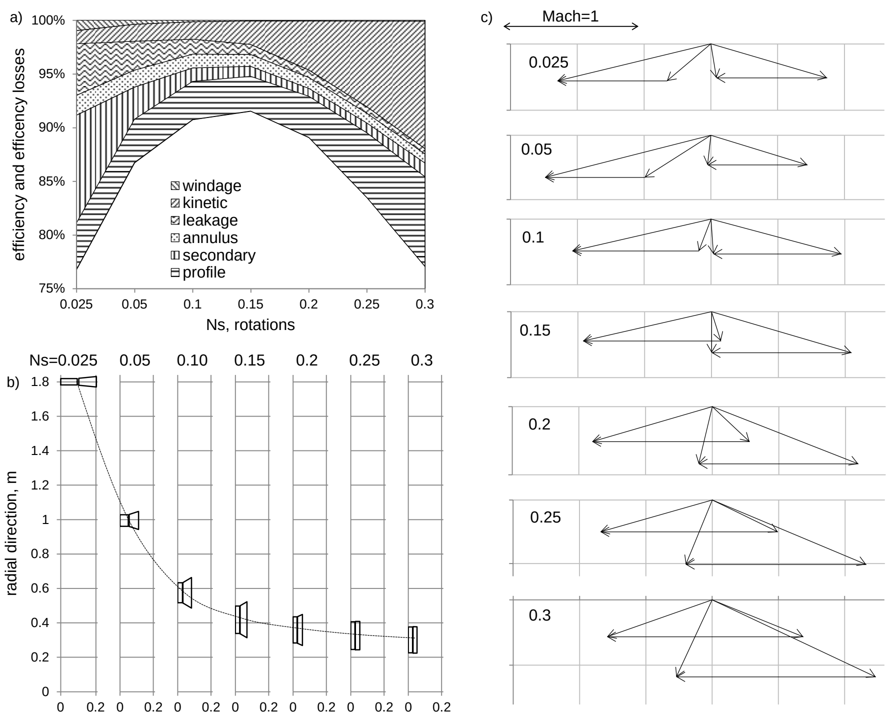

Figure 3 – Results of the parametric analysis varying Ns for a single stage turbine with SP=0.04 and Vr=4: a) Efficiency losses breakup, b) blade profiles and c) velocity diagrams. Mach numbers for inlet and outlet triangles are evaluated using the fluid SoS at stator and rotor outlet section respectively.

2.2. Effect of Vr at optimized Ns

Optimized results for single stage turbines, with different V_r and a fixed SP equal to 0.05, are presented in Fig 4. The breakdown of efficiency losses shows that the maximum attainable efficiency is a decreasing function of V_r. For really low volume ratios, namely values below twice than the $V_{r,crit} = 1.64$, velocity triangles are always subsonic, the load is limited and 50% reaction stages with high efficiency can be designed, h/D ratio is favourable, no flaring is required and both secondary and profile losses are small. Increasing V_r, Mach numbers greater than unit are obtained, in particular for v_1 and w_2 velocities, with an increase of profile losses. Converging-diverging stator nozzles are required for V_r above 5, while they are needed also for rotor blades for values beyond 10. For V_r greater than 20, a velocity close to the sonic one is obtained at w_1 vector with problems of shock waves at rotor inlet. The loss coefficient which takes into account this effect contributes to penalize the overall efficiency.

Velocity triangles become more and more distorted due to the necessity to handle higher volume flow variations and contextually maintaining a velocity vector v_2 close to the axial direction and limiting $\mathrm{M}w_1$. Increasing the volume ratio involves a higher isentropic enthalpy drop and optimized stages with a higher peripheral speed u and a larger mean diameter in order to limit the stage load. Solutions move toward impulse stages with very small blade heights and high secondary losses due to unfavorable h/D ratio and disk windage loss increases due to larger surface interested by the phenomena. In conclusion, adopting V_r higher than 5 for a small SP single stage turbine entails a strong limitation in the attainable efficiency and multistage turbines should be considered in order to contain the load on each stage.

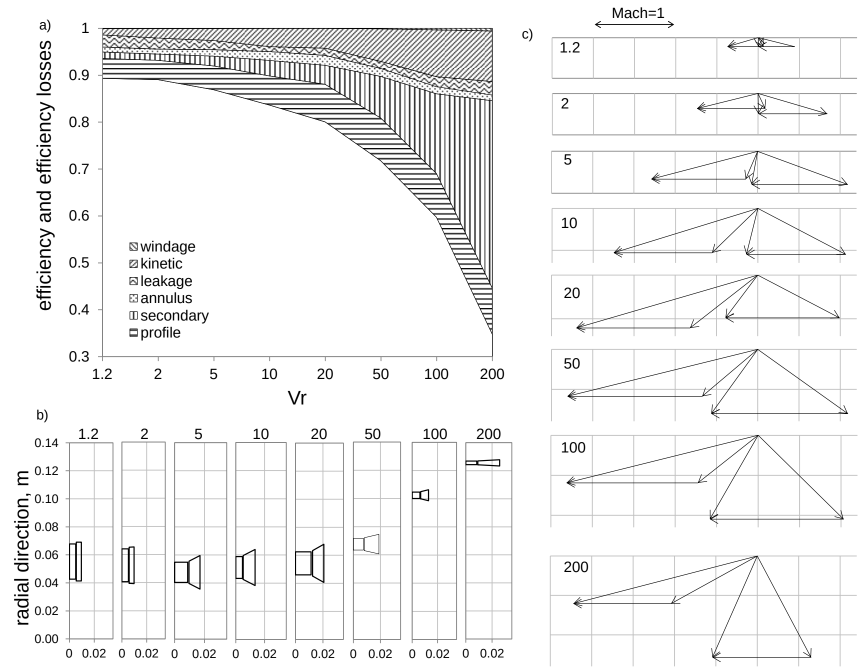

Figure 4 – Results of the parametric analysis for different single stage turbines having the same SP=0.05 m at the optimal specific speed: a) Efficiency losses breakup, b) blade profiles and c) velocity diagrams.

2.3. Effect of SP at optimized Ns

The last sensitivity analysis is realized for different single stage turbines with the same $V_r = 20$ but different SP and results are reported in Fig 5. Size parameter is varied from 0.02 m to 1 m which is a value representative of turbines close to the maximum size of normal ORC expanders. All the stages work with the same isentropic enthalpy drop but they notably differ in volumetric flow rate because of the quadratic dependence on the SP. Small size parameters lead to very small volume flow rates at turbine inlet section with a reduced passage area. Due to geometrical limits on minimum blade height and minimum δ_r/h ratio, the mean diameter gets smaller with an increase of rotational speed in order to maintain the optimal value of specific speed. These two effects result in a strong efficiency drop for small turbines with a considerable increase of secondary and leakage losses. Similar results are obtained for smaller V_r, even if the efficiency drop related to small SP decreases.

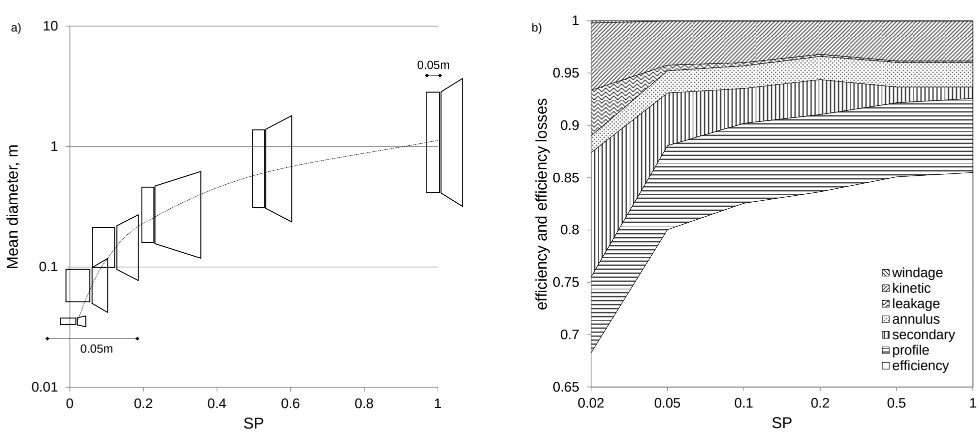

Figure 5 – Results of the parametric analysis for different single stage turbines having the same Vr=20 at the optimal specific speed: a) blade profiles and b) efficiency losses breakup.

4. MULTI STAGE TURBINE

In previous section, single stage turbines have been considered highlighting that their efficiency is strongly penalized by high volume ratios with even more marked reduction at small size parameters. In ORC field, multistage turbines are commonly adopted because they can achieve a higher efficiency exploiting the repartition of the whole volume flow variation on two or more stages. In this work only two and three stage turbines are considered because in most of the applications the benefit in adopting a higher number of stages is limited with an increase of component cost and a higher cost of electricity. For common applications, axial turbines are usually overhung with rolling bearing on the generator side of the shaft. This design allows an easy inspection of the turbine during maintenance operation and it is generally preferred even if it limits the number of stages to three because of rotodynamic issues. For turbines directly coupled to generator, a rotodynamic analysis performed by Exergy (Spadacini *et al*, 2013) shows that for a number of stages greater than three the natural frequencies of the turbine shaft get closer to 50 (or 60 Hz) with the risk of resonance during normal operation. A technical solution to increase the number of stages and the overall volume ratio without incurring in these problems is represented by radial outflow turbines.

The same analysis presented in the previous section is repeated for both two and three stage machines. The results, obtained optimizing the rotational speed for each turbine, are presented in terms of V_r and SP calculated for the overall expansion as though it is exploited by a single stage.

In Fig 6.a the comparison between the maximum efficiencies achievable for a single stage and a two stage turbine is reported while in Fig 6.b the increment of efficiency is displayed.

Increases of efficiency are not constant over the considered range of SP and V_r. The most relevant increments are obtained for high overall volume ratios and small SP. For V_r equal to 5 the efficiency increment is greater than 2 percentage points independently of SP value: an increment that usually justifies a more expensive device with the adoption of a two stage turbine.

Benefits are obviously larger for higher V_r and the efficiency increase reaches values above 10% for medium-small machines and volume ratios greater than 50. Last observation regards the possibility to extend the domain of solution: in particular for a SP lower than 0.02 m and V_r of 100 and 200 it is not possible to design a full-admission single stage turbine with a reasonable efficiency. The high load on such a small stage and the presence of geometric constrains entail a non-feasible execution of the optimization algorithm with variables values always outside of the efficiency losses correlation limits.

This problem does not arise for two stage turbines and the whole range of SP and V_r is explored.

Similar considerations can be done comparing two stages and three stage turbines. Results are reported in Fig 7.a and Fig. 7.b. The attainable efficiency increase is lower than in the previous case but, once again, notable advantages are highlighted for high volume ratios and small turbines. Finally, a graphical comparison of the performance maps for the single stage, the two stage and the three stage turbine are reported in Fig 6.c and 7.c.

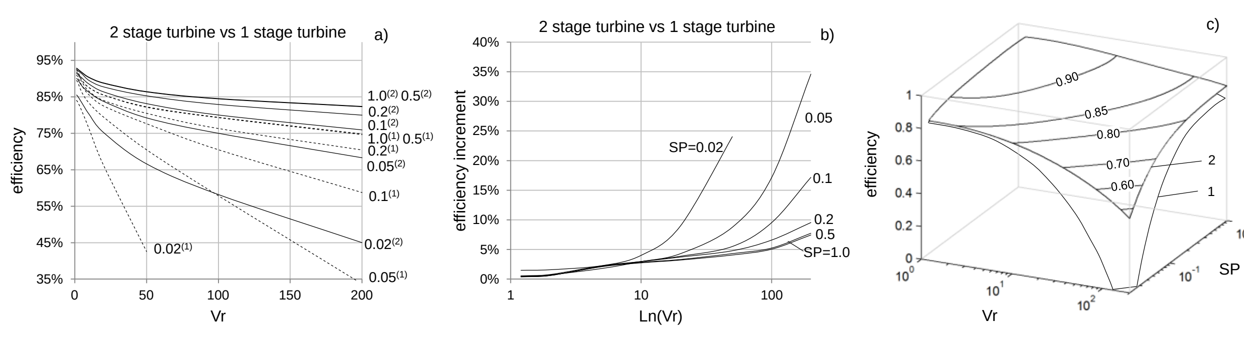

Figure 6 - Comparison between attainable efficiency adopting a two stage turbine instead of a single stage turbine (a) and corresponding efficiency increases (b). Graphical representation of maps of efficiency (c).

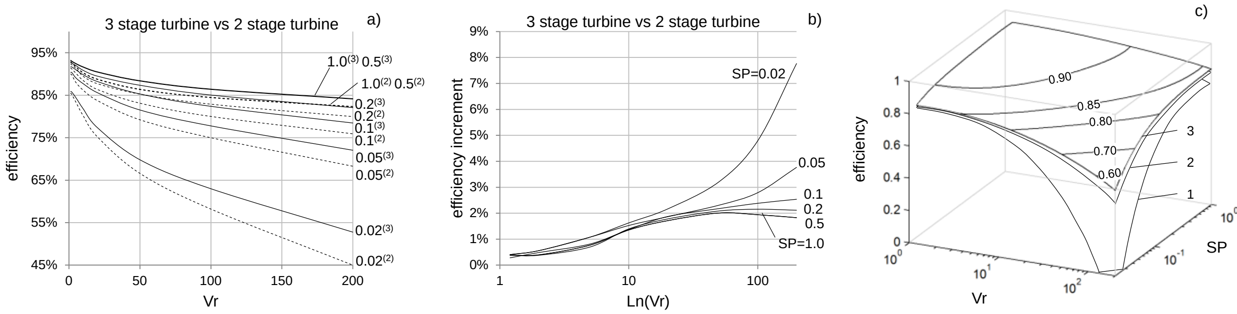

Figure 7 - Comparison between attainable efficiency adopting a three stage turbine instead of a two stage turbine (a) and corresponding efficiency increases (b). Graphical representation of maps of efficiency (c).

5. NUMERICAL CORRELATIONS OF EFFICIENCY

The set of data of maximum attainable turbine efficiency are regressed with a OLS regression performed in Gretl. The most suitable terms function of SP and V_r are selected and a process of exclusion of the less influencing ones is carried out in order to obtain the maximum value of the adjusted R^2 coefficient. All the proposed correlations have a functional form reported in eq 2 while the numerical values of the retrieved coefficients can be found in Table 1.

$$\eta = \sum_{i=0}^{15} A_i F_i \tag{2}$$

Table 1 - Regressed coefficients to be used in the correlation of turbine efficiency for single, two and three stage turbines

Stages number		1	2	3
n	F_i	A_i		
0	1	0.90831500	0.923406	0.932274
1	$\ln(SP)$	-0.05248690	-0.0221021	-0.01243
2	$\ln(SP)^2$	-0.04799080	-0.0233814	-0.018
3	$\ln(SP)^3$	-0.01710380	-0.00844961	-0.00716
4	$\ln(SP)^4$	-0.00244002	-0.0012978	-0.00118
5	Vr	-	-0.00069293	-0.00044
6	$\ln(Vr)$	0.04961780	0.0146911	-
7	$\ln(Vr)^2$	-0.04894860	-0.0102795	-
8	$\ln(Vr)^3$	0.01171650	-	-0.0016
9	$\ln(Vr)^4$	-0.00100473	0.000317241	0.000298
10	$\ln(Vr)\ \ln(SP)$	0.05645970	0.0163959	0.005959
12	$\ln(Vr)^2 \ln(SP)$	-0.01859440	-0.00515265	-0.00163
12	$\ln(Vr)\ \ln(SP)^2$	0.01288860	0.00358361	0.001946
13	$\ln(Vr)^3 \ln(SP)$	0.00178187	0.000554726	0.000163
14	$\ln(Vr)^3 \ln(SP)^2$	-0.00021196	-	-
15	$\ln(Vr)^2 \ln(SP)^3$	0.00078667	0.000293607	0.000211
Adjusted R^2		0.99790	0.99935	0.99954

6. MODEL VALIDATION

In this section an example about the capabilities of the proposed correlations is provided. In particular, the aim is underling how the volume ratio and the dimension of a real turbine stage can affect the turbine performance. To validate the proposed methodology two fluids (R125 and hexane) are considered and two turbine sizes (isentropic power of 5 MW and 250 kW) are analysed. Both fluids expand from a temperature of 155°C down to a pressure equal to the saturation at 30°C as representative of a low temperature heat source application with a cooling water condenser. A supercritical cycle is considered for R125 with a turbine inlet pressure of 36,2 bar while a saturated cycle is imposed for hexane according to its high critical temperature and the overhanging saturated vapour line which allows for a dry expansion. In Table 2 the main thermodynamic properties of the fluids are reported with other quantity of direct interest for the evaluation of turbine efficiency. Both R125 and hexane show real gas effects at turbine inlet with a compressibility factor equal to 0.85 and 0.79 respectively while a behaviour closer to ideal gas is founded at turbine discharge with values of 0.91 and 0.99. The overall expansion coefficients (γ) are different from the value assumed for the ideal gas in the previous sections and are equal to 1.10 and 1.06. The two turbine stage differ in both the Vr and the SP. The expansion of R125 shows a very limited variation of density because of the high condensation pressure and the high degree of superheating. On the contrary hexane according to the corresponding state principle (Poling and Prausnitz , 2000) has a lower condensing pressure (a vacuum pump is required at the condenser to remove air leakage) and a larger volume flow rate variation across the turbine. Hexane mass flow rate is lower than the R125 one but the SP is larger because of the very low density at turbine outlet section. Using the correlation of efficiency previously described the turbine efficiency is calculated: R125 turbine shows a very high (90%) efficiency while for hexane the presence of supersonic flows and limitation of flaring angles play a detrimental role in the final efficiency which is close to 83%. In this case the positive effect of a larger SP is not sufficient to compensate the difficulties in realizing a high Vr stage. Reducing the size of the turbines the presence of geometrical constraints (minimum o/s and maximum o/b) bound the final solution and leads to a decrement of about 3 point of efficiency confirming the overall trend presented in previous analyses.

Table 2 – Characteristics of the considered fluids and main results of the analysis

	R125		hexane	
critical properties				
MM	120.02		86.18	
T_{crit}, °C	66.02		234.67	
p_{crit}, bar	36.18		30.34	
expansion data				
W_{is}, kW	5000	250	5000	250
p_{in}, bar	36.200		8.290	
p_{out}, bar	15.685		0.250	
$\Delta T_{sh}, °C$	88.980		0	
m, kg/s	237.71	11.89	40.78	2.04
$V_{out,is}$, m^3/s	3.79	0.19	55.24	2.76
specific parameters				
V_r	2.293	2.293	34.389	34.389
SP, m	0.162	0.036	0.397	0.089
$\eta_{is}^{\text{correlation}}$	0.903	0.877	0.833	0.799
Axtur results				
η_{is}^{Axtur}	0.907	0.872	0.828	0.795
RPM	6000	31000	5500	2800
Ns	0.111	0.129	0.104	0.118
D, m	0.420	0.086	0.900	0.180

The four turbines are hence designed in Axtur considering real fluid properties (i.e. compressibility effects, Mach numbers, etc.) and optimizing the rotational speed. It is possible to highlight a good accordance among the efficiencies calculated with the two methods confirming the validity of the proposed approach. The optimal rotational speed, the optimal Ns and the resulting optimal diameter are reported in table as well.

7. CONCLUSIONS

In this work three correlations of performance are provided for axial turbines with a maximum number of stages equal to three. This study aims to complete the work done in previous publications about the estimation of maximum efficiency attainable with 1D optimization techniques. The correlations can be used for a preliminary estimation of turbine performance in the numerical optimization of ORC even if the results are affected by inaccuracy mainly related to the quality of efficiency losses correlations and the simplified volumetric behavior assumed in the generation of the performance maps.
On the basis of the present analyses the following conclusions can be addressed:

- Rotational speed must be always optimized since relevant efficiency decrements are highlighted for values lower and higher than the optimal one. The use of slow generators with more than two couple of poles is recommended for large turbines while a gearbox or a power electronic system is required for small size machines having an optimal rotational speed higher than 3000 RPM;
- Geometrical similarity cannot be always verified because of the presence of geometrical constraints related to blade machining and limits of loss correlations. Decreasing the size of the turbine the maximum efficiency is reduced mainly because of the increasing of profile and leakage losses;
- Isentropic volume ratio strongly affects stage design and efficiency. A single stage turbine with high Vr is penalized because of the presence of big flaring angles, supersonic flows and high kinetic losses and it is advantageous to split the expansion in two or more stages
- The contemporary presence of small SP and high VR strongly penalizes the attainable efficiency of single stage machines and suggests the adoption of multi-stage solutions.

NOMENCLATURE

Variables

A	coefficients for the efficiency correlations	(-)
b	axial chord	(m)
β	pressure ratio	(-)
η	efficiency	(-)
F	terms of the efficiency correlations	(-)
φ_E	kinetic energy recovery factor	(-)
γ	ratio of specific heats	(-)
k_{is}	Isentropic load coefficient	(-)
h	enthalpy or blade height	(m)
h/D	blade height-blade mean diameter ratio	(-)
m	mass flow rate	(kg/s)
Ns	specific speed	(-)
o	blade channels throat	(m)
p	pressure	(bar)
r	radius	(m)
ρ	density	(kg/m^3)
s	blade step	(m)

SP	Size Parameter	(m)
T	temperature	(°C)
V	volume flow rate	(m^3/s)
v	absolute velocity	(m/s)
u	mean peripheral speed	(m/s)
Vr	volume ratio	(-)
W	power	(kW)
w	relative velocity	(m/s)

Subscripts

crit	critical property
eva	evaporation
cond	condensation
is	isentropic
re	real
T-S	Total to Static

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